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Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Graph Theory and Combinatorics

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions, selecting at least TWO full questions from each part.

PART - A

- 1 a. Prove that in any graph the number of vertices of odd degree is even. (06 Marks)
 b. Define isomorphism. Prove that the following graphs are isomorphic: (07 Marks)

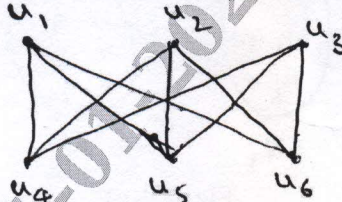


Fig.Q.1(b) (i)

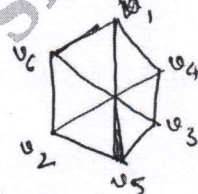


Fig.Q.1(b) (ii)

- c. Discuss Konigsberg problem and give the solution of the problem. (07 Marks)
- 2 a. Define planar graph, show that the graph K_5 is non planar graph. (06 Marks)
 b. Prove that a connected graph with 'n' vertices and on edges has exactly $m-n+2$ regions. (07 Marks)
 c. Find the chromatic polynomial and chromatic number of the graph Fig.Q.2(c). If four colours are used in how many ways can the graph be properly coloured (07 Marks)

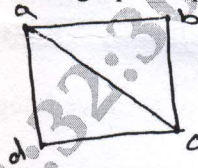


Fig.Q.2(c)

- 3 a. Prove that a tree with n vertices has n-1 edges. (06 Marks)
 b. Define spanning tree and find the spanning trees of the graph (Fig.Q.3(b)). (07 Marks)

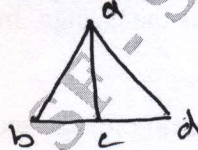


Fig.Q.3(b)

- c. Construct the optimal prefix code for the message "ROAD IS GOOD". Indicate the code. (07 Marks)

- 4 a. Using Kruskal's algorithm, find a minimal spanning tree for the weighted graph shown in the following Fig.Q.4(a). (06 Marks)

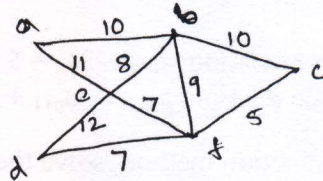


Fig.Q.4(a)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- b. For the network shown in Fig.Q.4(b), find the capacities of all cut-sets between the vertices 'a' and 'd' and hence find maximum flow between a and d. (07 Marks)

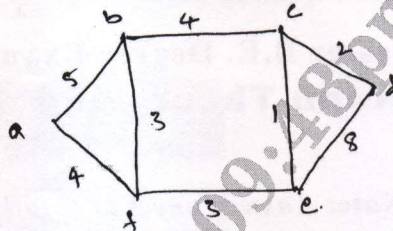


Fig.Q.4(b)

- c. Show that the following graph has a complete matching from V_1 to V_2 . Obtain two complete matching (Fig.Q.4(c)). (07 Marks)

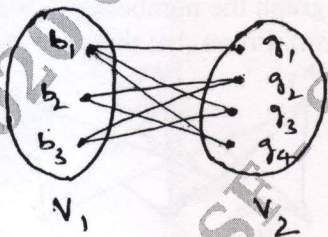


Fig.Q.4(c)

PART - B

- 5 a. Find the number of permutations of the letters of the following words:
 i) ENGINEERING ii) MATHEMATICS (06 Marks)
- b. Find the co-efficiency
 i) $w^3 x^2 z^2$ in the expansion of $(2w-x+3y-2z)^8$
 ii) xyz^{-2} in the expansion of $(x-y+3z^{-1})^4$. (07 Marks)
- c. Define catalan number. In how many ways can one arrange three 1's and three -1's, so that all six partial sums (starting from first) are non negative? List all the arrangements. (07 Marks)
- 6 a. Determine the number of positive integers 'n' such that $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. (06 Marks)
- b. Define derangement. In how many ways can the integers 1, 2, 3, 4, 5 be deranged? List those derangements where the first three numbers are 1, 2 and 3 in some order. (07 Marks)
- c. An apple, a banana, a mango and an orange are to be distributed to four boys B_1, B_2, B_3 and B_4 . The boys B_1 and B_2 do not wish to have the apple, the boy B_2 does not want the banana or mango and B_4 returns the orange. In how many ways the distribution can be made so that no boy is displeased? (07 Marks)
- 7 a. Find the generating function for the sequences:
 i) $1^2, 2^2, 3^2, 4^2, \dots$
 ii) $1, 2, 2^2, 2^3, 2^4, \dots$ (06 Marks)
- b. In how many ways can we distribute 24 pencils to four children, so that each child gets at least 3 pencil but not more than 8. (07 Marks)
- c. Define exponential generating function, using exponential generating function find the number of ways in which 5 of the letters in the word CALCULUS be arranged. (07 Marks)
- 8 a. Solve the recurrence relation $a_{n+1} - 3a_n = 5 \times 7^n, n \geq 0$, with $a_0 = 2$. (06 Marks)
- b. Solve the recurrence relation $a_{n+2} + 4a_{n+1} + 4a_n = 7, n \geq 0$, given that $a_0 = 1, a_1 = 2$. (07 Marks)
- c. Using generating function method, solve the recurrence relation $a_{n+2} - 3a_{n+1} + 2a_n = 0$ for $n \geq 0$, given $a_0 = 1$ and $a_1 = 6$. (07 Marks)
